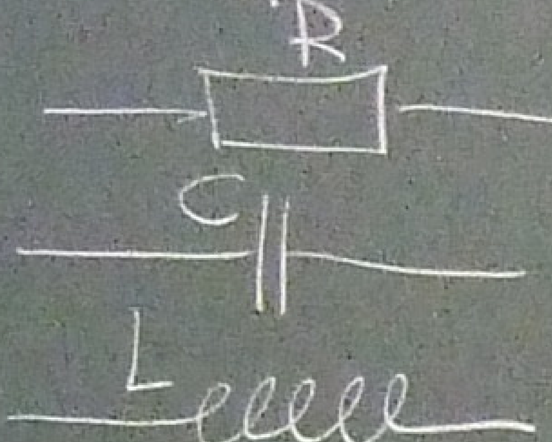
Elemente von Stromkreisen



Widerstand R $\quad U = R\,I \quad \dot{U}_R = R\cdot\dot{I}$

Kapazität C $\quad U = Q/C \quad \dot{U}_C = \frac{I}{C}$

Induktivität L $\quad U = L\cdot\dot{I} \quad \dot{U}_L = L\,\ddot{I}$

Einheiten: $R = \frac{V}{A} = \Omega \qquad L = \frac{Vs}{A} = Hy$ (Henry)

$C = \frac{C}{V} = F$ (arad)

Relevante Zeitskalen

Ansätze: $I = \sin\omega t$

$\omega = 2\pi f$

$\dot{I} = \omega\cdot\cos\omega t$

$\ddot{I} = -\omega^2 \sin\omega t$

RC-Glied

$|\dot{U}_R| = |\dot{U}_C|$

$R\cdot\omega = \frac{1}{C} \rightsquigarrow f = \frac{1}{2\pi R\cdot C}$

RL-Glied

$|\dot{U}_R| = |\dot{U}_L|$

$R\omega = L\omega^2$

$\rightsquigarrow f = \frac{R}{2\pi L}$

CL-Glied

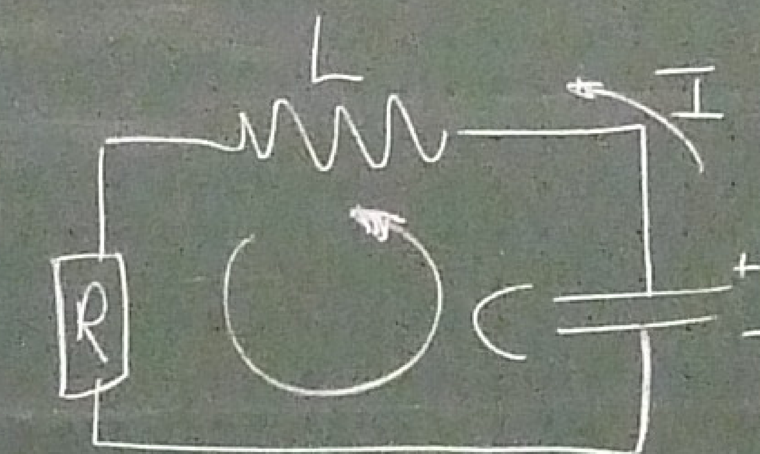
$|\dot{U}_C| = |\dot{U}_L|$

$\frac{1}{C} = \omega^2 L$

$\omega = 2\pi f = \sqrt{\frac{1}{CL}} \rightsquigarrow f = \frac{1}{2\pi\sqrt{C\cdot L}}$

RLC Stromkreis:

Das elektrische Schwingkreis



$U_R = I\cdot R$

$U_C = Q/C$

$U_L = -L\frac{dI}{dt}$

Kirchhoff: $U_L + U_R + U_C = 0$

$L\dot{I} + RI + Q/C = 0 \quad \Big|\ \frac{d}{dt\cdot L}$

$\ddot{I} + \frac{R}{L}\dot{I} + \frac{I}{LC} = 0$

Analog $\ddot{x} + \frac{k}{m}\dot{x} + \frac{D}{m}x = 0$

Gedämpfter mechanischer Oszillator

Feder

Dämpfung Reibung

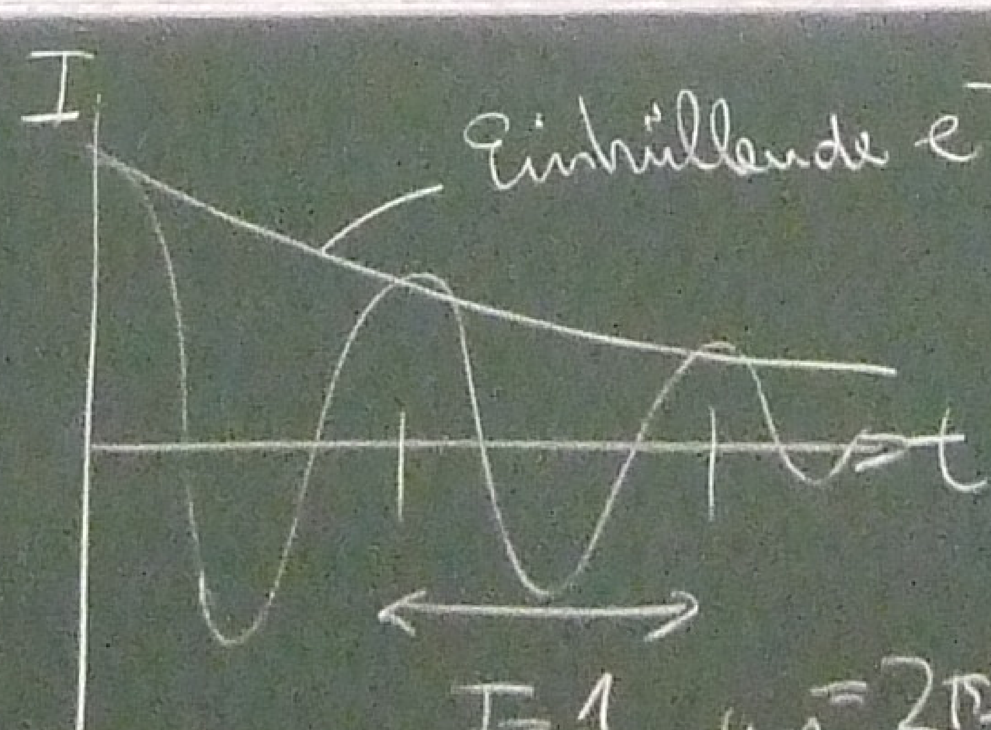
Massenträgheit

Lösung:

$I(t) = I_0\, e^{-\beta t} \cos\omega_0 t$

$\beta = \frac{R}{2L}\ ; \quad \omega_0^2 \approx \frac{1}{LC}$

wenig R



$T = \frac{1}{f}\,,\ \omega_0 = 2\pi f$

$L = 630\,Hy$

$R = 280\,\Omega$ (in der Spule)

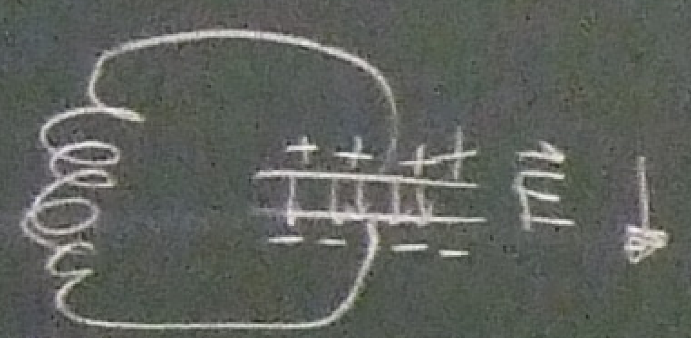
$C = 16\,\mu F$

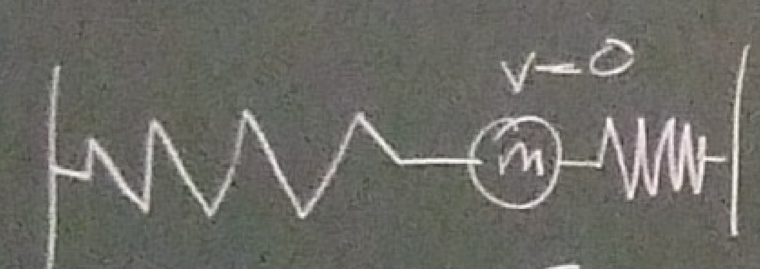
$\beta = \frac{R}{2L} = 0{,}22\,1/s$

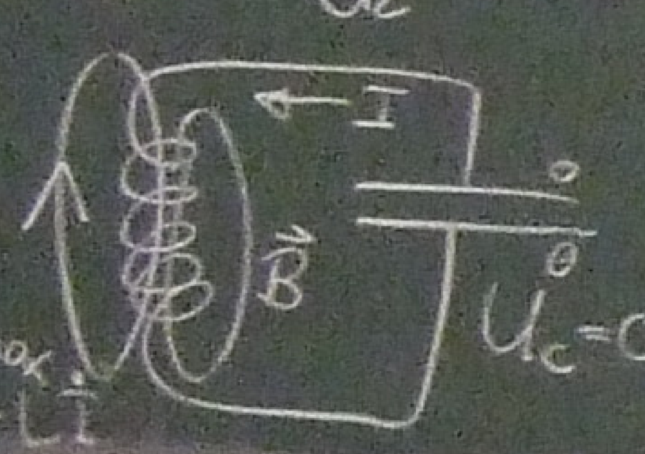
$\tau = \frac{1}{\beta} = 4{,}5\,s$

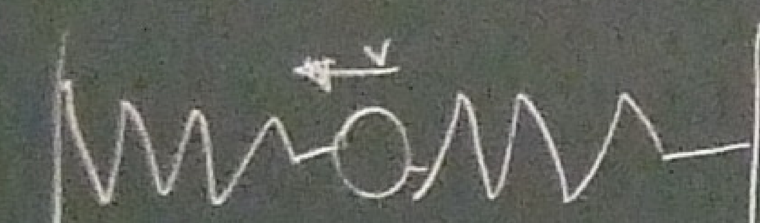
$f_0 = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} = 1{,}58\,Hz$

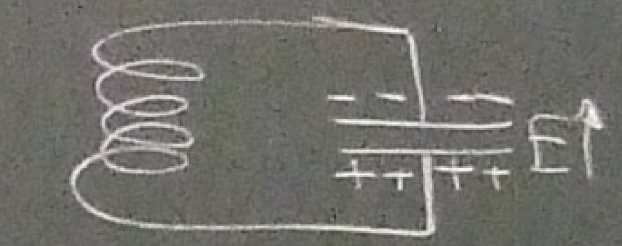
Analogie mit mechanischen Pendel ($R=0$)

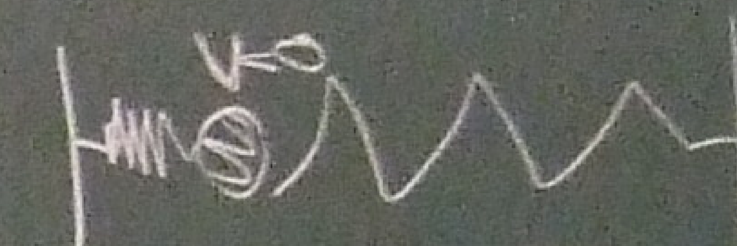


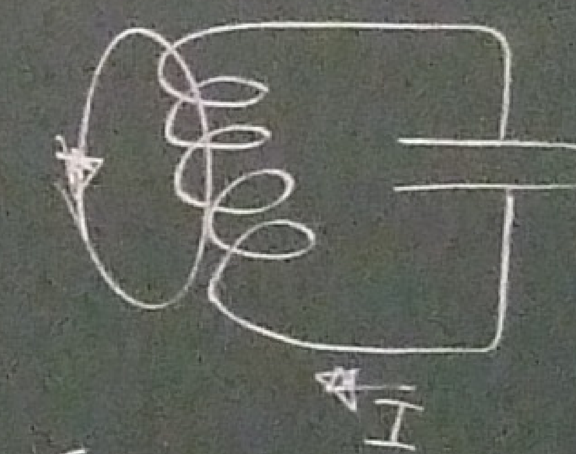


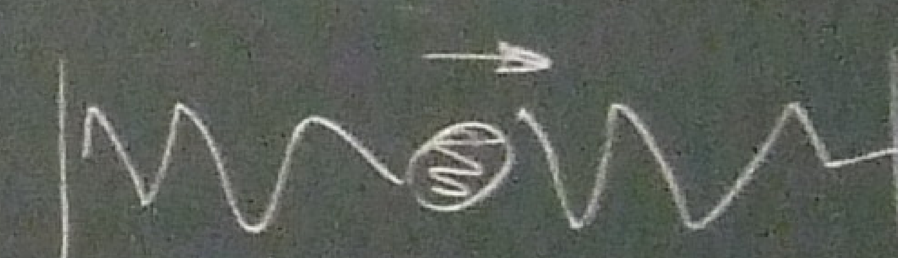










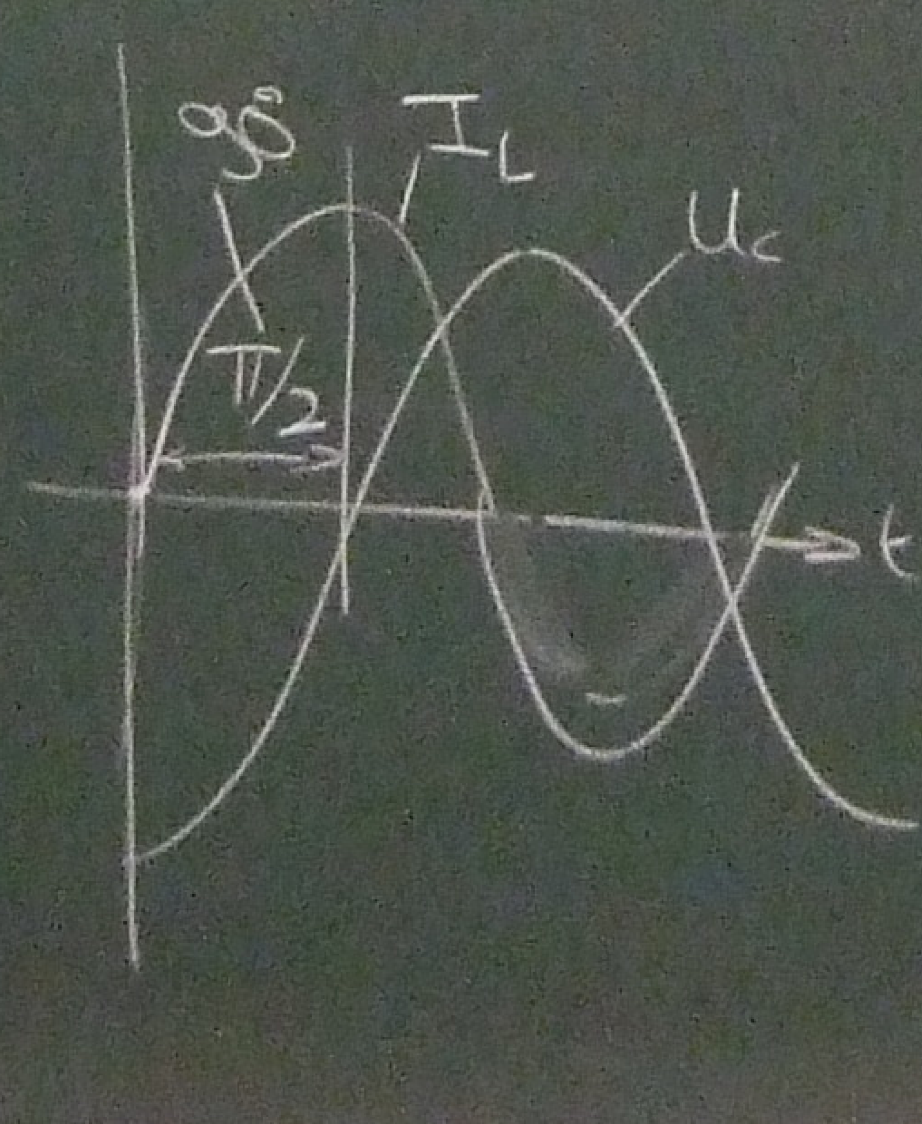


und zurück zum Anfang

Strom durch $L = I_0 \sin \omega t$

Spannung an C: $U_C = \frac{q}{C} = \frac{\int I\,dt}{C}$

$= \frac{-I_0}{\omega C} \cos \omega t$

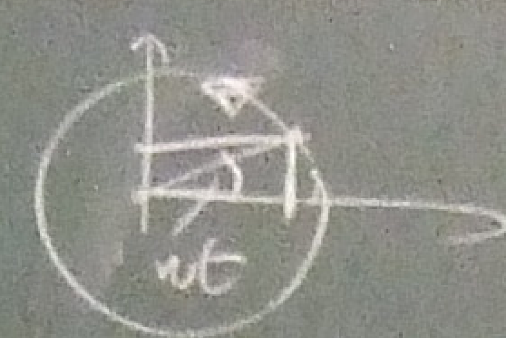


$|U_R| = |U_C|$

sin cos

$I = I_0 \sin \omega t$

$e^{i\omega t} = \cos \omega t + i \sin \omega t$



Gütefaktor

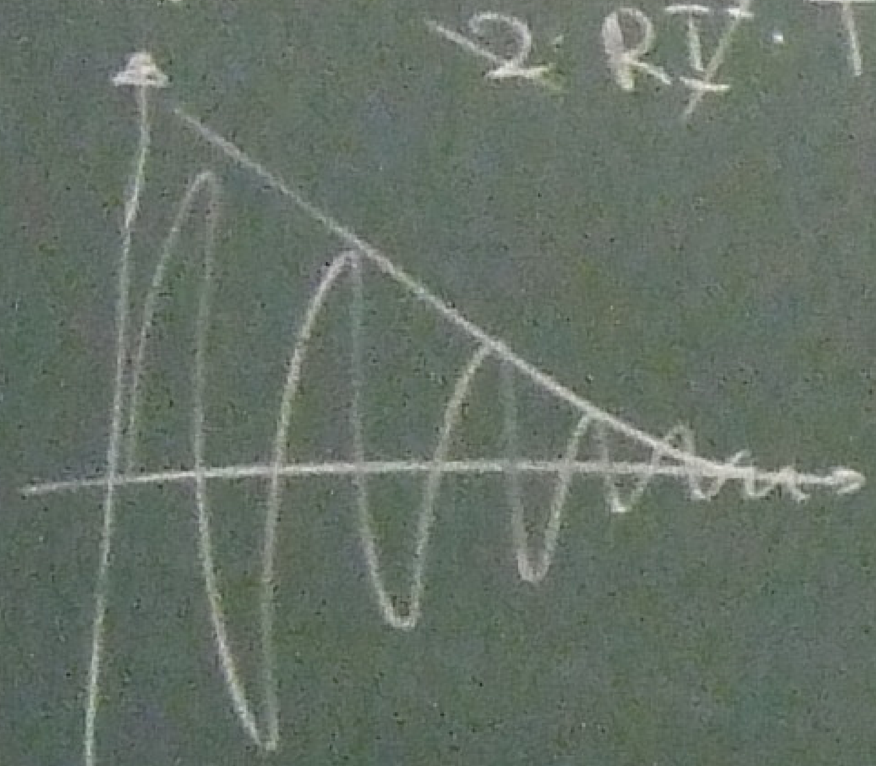
$$Q = 2\pi \cdot \frac{\text{Gespeicherte Energie}}{\text{In einer Periode abgegebene Energie}}$$

am Beispiel Schwingkreis:

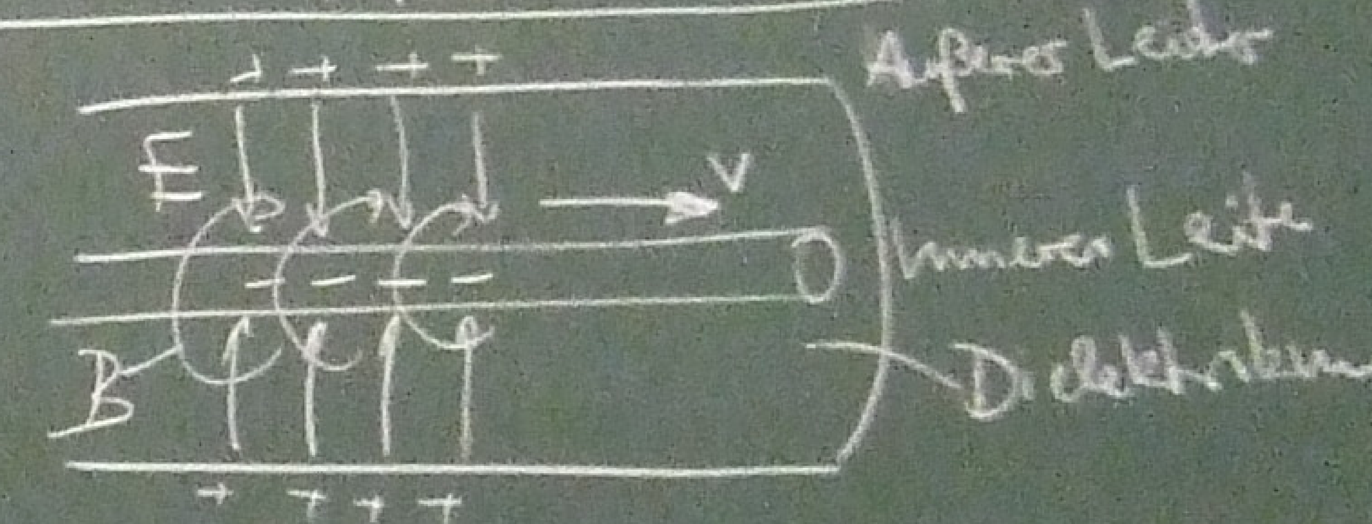
- Gespeicherte Energie: $W = \frac{LI^2}{2}$
- Abgegebene Energie in einer Periode T: $\dot{W} \cdot T = RI^2 \cdot T$ (Joule'sche Wärme)

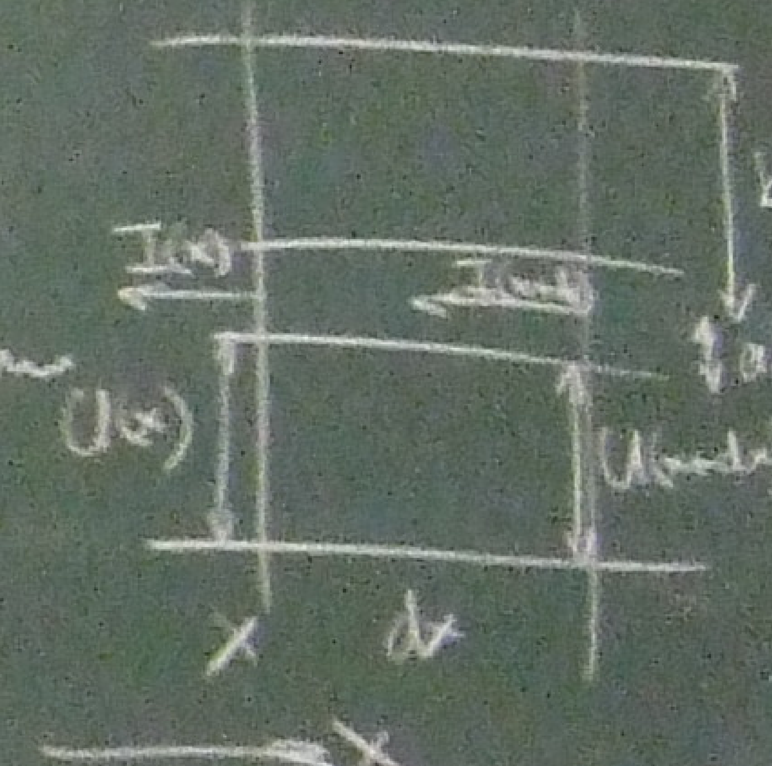
$$Q = 2\pi \cdot \frac{LI^2}{2 \cdot RI^2 \cdot T} = \frac{\pi L}{R \cdot T} = 11$$

$L = 630\,\text{Hy}$,
$R = 280\,\Omega$
$C = 16\,\mu F$.



Ausbreitung elektrischer Wellen bzw. Signale in Leitern: Koaxialkabel.





Auf Länge bezogen: $C = C_0/l$. Kap. pro Länge

$L = L_0/l$

Induktionsgesetz: $dU = -L\,dx \frac{\partial I}{\partial t} \rightsquigarrow \frac{\partial U}{\partial x} = -L \frac{\partial I}{\partial t}$ $\quad \Big| \; \partial/\partial t$ oder $\frac{\partial}{\partial x}$

Kapazität: $-dI = d\left(\frac{\partial Q}{\partial t}\right) = C\,dx \frac{\partial U}{\partial t} \rightsquigarrow \frac{\partial I}{\partial x} = -C \frac{\partial U}{\partial t}$ $\quad \Big| \; \partial/\partial x$

$$\frac{\partial^2 I}{\partial x^2} = L C \frac{\partial^2 I}{\partial t^2} \quad \text{oder} \quad \frac{\partial^2 U}{\partial x^2} = L \cdot C \frac{\partial^2 U}{\partial t^2}$$

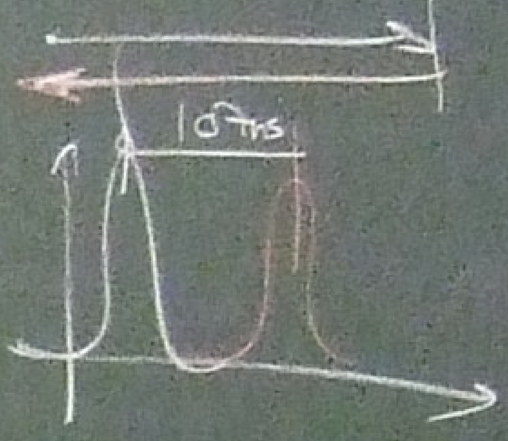
$U(x,t)$

Wellengleichungen mit einer Ausbreitungsgeschwindigkeit von $v = \frac{1}{\sqrt{L \cdot C}}$

Zylinderkondensator

$$C_0 = \frac{Q}{U} = \frac{Q}{\int E\,dr} = \frac{Q}{\int_a^b \frac{Q\,dr}{l\,2\pi\varepsilon\varepsilon_0 r}} = \frac{2\pi\varepsilon\varepsilon_0}{\ln b/a}\,l \underset{\text{gemessen}}{=} \frac{100\,pF\cdot 1m}{1m} \qquad C = C_0/l = 100\,pF/m$$

$$v = \frac{20\,m}{107\,ns} = 1{,}87\cdot 10^8\,\frac{m}{s}$$



Leitungsinduktivität (längere Disk)

$$L_0 = \frac{\mu_0}{2\pi}\ln b/a\cdot l \longrightarrow L = \frac{\mu_0}{2\pi}\ln b/a \rightsquigarrow L\cdot C = \varepsilon\varepsilon_0\mu_0 = \frac{1}{v^2} = \frac{\varepsilon}{c^2}$$

$$\rightsquigarrow c = \sqrt{\varepsilon}\cdot v$$ Später: Brechungsindex

Reflexion am Leiterende

Lösung der Wellengleichung:

$$U = U_0\sin(\omega t - kx)$$

Wegen $\frac{\partial I}{\partial x} = -C\frac{\partial U}{\partial t} \rightsquigarrow I = -C\int\frac{dU}{dt}\,dx$

$$= -C\omega\int U_0\cos(\omega t - kx)\,dx$$

$$I = \underbrace{\frac{C\omega U_0}{k}}\cdot\sin(\omega t - kx)$$

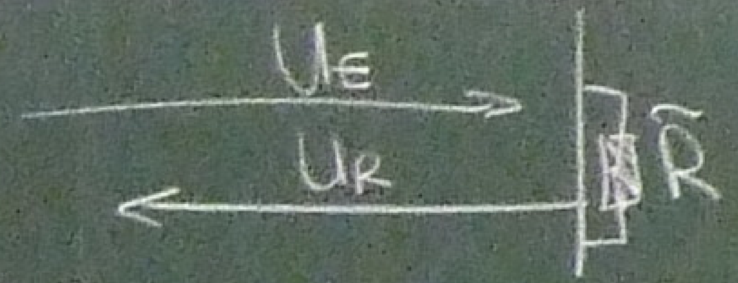
Als Wellenwiderstand R definiert man den Eingangswiderstand einer ∞-langen Leitung

$$R = \frac{U}{I} = \frac{k}{\omega C} \underset{v=\frac{\omega}{k}}{=} \frac{1}{vC} = \sqrt{\frac{L}{C}} = 50\,\Omega$$

$$L = 2{,}5\cdot 10^{-7}\,Hy/m$$

Einfallende Welle $U_E = U_{E0}\sin(\omega t - kx)$

Refl. Welle $U_R = U_{R0}\sin(\omega t + kx)$



$$U(x,t) = U_E + U_R$$
$$R\cdot I = U_E - U_R$$

Definiert man Reflexionsfaktor $\rho = \frac{U_{R0}}{U_{E0}}$

Wenn mit $\hat R$ bei x=0 abgeschlossen

$$\frac{U(0,t)}{I(0,t)} = \tilde R = \frac{U_{E0}+U_{R0}}{U_{E0}-U_{R0}}\cdot R \rightsquigarrow \rho = \frac{\tilde R - R}{\tilde R + R}$$

1) $\hat R = \infty$ $\quad \rho = 1$: Reflex offenes Ende

2) $\hat R = R$ $\quad \rho = 0$: Abschluß mit R: kein Reflex

3) $\hat R = 0$ $\quad \rho = -1$ Reflex am Kurzschluß